# The 20th International Workshop
on Hermitian Symmetric Spaces and Submanifolds
& The 12th RIRCM-OCAMI Joint Differential Geometry Workshop

**July 26 (Tuesday), 2016**

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<tr>
<th>Time</th>
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| 11:30–12:30   | **Chair** Professor Juan de Dios Perez  
Recent results about Jacobi operator for real hypersurfaces in complex two plane Grassmannians  
Eunmi Pak* (Kyungil University & RIRCM, Korea) and  
Young Jin Suh (Kyungpook National University & RIRCM)  
The parallelism for real hypersurfaces in complex Grassmannians of rank two  
Hyunjin Lee* (Research Institute of Real and Complex Manifolds of KNU (RIRCM), Korea) and  
Young Jin Suh (Kyungpook National University & RIRCM, Korea) |
| 12:30–14:00   | Lunch Time                                                                                  |
| 14:00–15:00   | **Chair** Professor Byung Hak Kim  
Hypersurfaces in complex hyperbolic two-plane Grassmannians with GTW Reeb Lie derivative structure  
Gyu Jong Kim* (Kyungpook National University, Korea) and  
Young Jin Suh (Kyungpook National University & RIRCM, Korea)  
Comparison on classification results of real hypersurfaces in compact and noncompact complex two-plane Grassmannians  
Young Jin Suh (Kyungpook National University & RIRCM, Korea) and  
Changhwa Woo* (Kyungpook National University, Korea) |
| 15:00–15:10   | Break Time                                                                                  |
| 15:10–16:10   | **Chair** Professor Alfonso Romero  
Characterizations of a Clifford hypersurface in a unit sphere  
Keomkyo Seo (Sookmyung Women’s University, Korea)  
On the pointwise slant submanifolds  
Kwang-Soon Park (University of Seoul, Korea) |
| 16:10–16:30   | Break Time                                                                                  |
| 16:30–18:00   | **Chair** Professor Young Jin Suh  
CR geometry on contact manifolds  
Jong Taek Cho (Chonnam National University, Korea)  
Quaternionic projective spaces and 7-sphere  
Jae-Hyouk Lee (Ewha Womans University, Korea)  
Gromov-Witten invariants on the products of almost contact manifolds  
Yong Seung Cho (Ewha Womans University, Korea) |
# The 20th International Workshop on Hermitian Symmetric Spaces and Submanifolds & The 12th RIRCM-OCAMI Joint Differential Geometry Workshop

## July 27 (Wednesday), 2016

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<tr>
<td>09:00~12:40</td>
<td><strong>Chair</strong> Professor Yoshihiro Ohnita</td>
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</table>
| 09:00~10:00 | Integral Geometry along flat subvarieties in symmetric spaces of compact type (I)  
|           | Eric Grinberg (University of Massachusetts Boston, USA)                 |
| 10:10~11:10 | Hamiltonian non-displaceability of the Gauss images of isoparametric hypersurfaces (I)  
|           | Reiko Miyaoka (Tohoku University, Japan)                                |
| 11:00~11:20 | Break Time                                                              |
| 11:20~12:20 | LVMB manifolds and transverse Kahler foliations                         
|           | Jin Hong Kim (Chosun University, Korea)                                 |
| 12:20~12:40 | Opening Address & Group Photo I                                         |
| 12:40~14:00 | Lunch Time                                                              |
| 14:00~15:30 | **Chair** Professor Eric Grinberg                                        |
| 14:00~14:45 | On Floer homology of the Gauss images of isoparametric hypersurfaces    
|           | Yoshihiro Ohnita (OCAMI & Osaka City University, Japan)                 |
| 14:45~15:30 | On Bonnesen-style inequalities for mixed volumes                         
|           | Jiazu Zhou (Southwest University, China)                                |
| 15:30~17:00 | Discussion Time                                                         |
| 17:00~18:00 | **Chair** Professor Jiazu Zhou                                          |
| 17:00~18:00 | Quantitative rigidity on manifolds of Ricci curvature bound below (I)   
|           | Xiaochun Rong (Rutgers, The State University of New Jersey, USA)        |
The 20th International Workshop  
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July 28 (Thursday), 2016

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<th>Institution(s)</th>
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<tr>
<td>09:00~11:10</td>
<td>Chair Professor Xiaochun Rong</td>
<td>Integral Geometry along flat subvarieties in symmetric spaces of compact type (II)</td>
<td>Eric Grinberg (University of Massachusetts Boston, USA)</td>
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<tr>
<td></td>
<td></td>
<td>Hamiltonian non-displaceability of the Gauss images of isoparametric hypersurfaces (II)</td>
<td>Reiko Miyaoka (Tohoku University, Japan)</td>
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<tr>
<td>11:10~11:20</td>
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<td>Break Time</td>
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<tr>
<td>11:20~12:40</td>
<td>Chair Professor Jiazu Zhou</td>
<td>The classical Calabi-Bernstein theorem: history and new approaches</td>
<td>Alfonso Romero (University of Granada, Spain)</td>
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<tr>
<td>12:40~14:00</td>
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<td>Lunch Time</td>
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<tr>
<td>14:00~15:30</td>
<td>Chair Professor Keom Kyo Seo</td>
<td>Sequences of maximal antipodal sets of oriented real Grassmann manifolds II</td>
<td>Hiroyuki Tasaki (University of Tsukuba, Japan)</td>
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<td>Maximal antipodal subgroups of the automorphism groups of compact Lie algebras</td>
<td>Makiko Tanaka* (Tokyo University of Science, Japan) and Hiroyuki Tasaki (University of Tsukuba, Japan)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Biharmonic homogeneous submanifolds in compact symmetric spaces</td>
<td>Shinji Ohno (Osaka City University Advanced Mathematical Institute (OCAMI), Japan), Takashi Sakai* (Tokyo Metropolitan University, Japan) and Hajime Urakawa (Tohoku University, Japan)</td>
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<tr>
<td>15:30~17:00</td>
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<td>Discussion Time</td>
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<tr>
<td>17:00~18:00</td>
<td>Chair Professor Reiko Miyaoka</td>
<td>Quantitative space from rigidity under lower Ricci curvature bound (II)</td>
<td>Xiaochun Rong (Rutgers, The State University of New Jersey, USA)</td>
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<th>Session 2</th>
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<tr>
<td>14:00~15:30</td>
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# The 20th International Workshop on Hermitian Symmetric Spaces and Submanifolds & The 12th RIRCM-OCAMI Joint Differential Geometry Workshop

## July 29 (Friday), 2016

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<tbody>
<tr>
<td>09:00~11:15</td>
<td>Chair Professor Juan de Dios Perez</td>
<td>Integral Geometry along flat subvarieties in symmetric spaces of compact type (III)</td>
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<tr>
<td></td>
<td></td>
<td>Eric Grinberg (University of Massachusetts Boston, USA)</td>
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<tr>
<td>10:15~11:15</td>
<td>Hamiltonian non-displaceability of the Gauss images of isoparametric hypersurfaces (III)</td>
<td>Reiko Miyaoka (Tohoku University, Japan)</td>
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<tr>
<td>11:15~11:30</td>
<td>Break Time</td>
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<tr>
<td>11:30~12:30</td>
<td>Chair Professor Young Jin Suh</td>
<td>Spacelike surfaces in GRW spacetimes whose mean curvature function satisfies a nonlinear inequality</td>
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<td>Alfonso Romero (University of Granada, Spain)</td>
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<tr>
<td>12:30~14:00</td>
<td>Lunch Time</td>
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<tr>
<td>14:00~15:30</td>
<td>Chair Professor Jong Taek Cho</td>
<td>Almost complex structures on four-dimensional neutral manifolds</td>
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<td>Yasuo Matsushita (Osaka City University Advanced Mathematical Institute (OCAMI), Japan)</td>
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<tr>
<td>14:30~15:00</td>
<td>Weyl, projective and conformal semi-symmetric complex hypersurfaces in the semi-Kaehler space forms</td>
<td>Young Suk Choi (Kyungpook National University, Korea)</td>
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<tr>
<td>15:00~15:30</td>
<td>On 3-dimensional real hypersurfaces in complex space forms</td>
<td>Mayuko Kon (Shinshu University, Japan)</td>
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<tr>
<td>15:30~17:00</td>
<td>Discussion Time</td>
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<tr>
<td>17:00~18:00</td>
<td>Chair Professor Alfonso Romero</td>
<td>Inequalities for algebraic Casorati curvatures and their applications</td>
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<td>Mukut Mani Tripathi (Banaras Hindu University, India)</td>
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## Session 1

### [Session 1]

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<th>Chair Professor Jin Hong Kim</th>
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<tbody>
<tr>
<td>14:00~15:30</td>
<td></td>
<td>The Schwarz lemma for super-conformal maps</td>
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<td>Katsuhiro Moriya (University of Tsukuba, Japan)</td>
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<tr>
<td>14:30~15:00</td>
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<td>Transversally complex submanifolds of a quaternion projective space</td>
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<td>Kazumi Tsukada (Ochanomizu University, Japan)</td>
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<tr>
<td>15:00~15:30</td>
<td></td>
<td>A construction of weakly reflective submanifolds in compact symmetric spaces</td>
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<td></td>
<td>Shinji Ohno (Tokyo Metropolitan University &amp; OCAMI, Japan)</td>
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# The 20th International Workshop on Hermitian Symmetric Spaces and Submanifolds & The 12th RIRCM-OCAMI Joint Differential Geometry Workshop

**July 30 (Saturday), 2016**

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<tr>
<td>10:00~11:00</td>
<td>Derivatives on real hypersurfaces in nonflat complex space forms</td>
<td>Juan de Dios Perez (University of Granada, Spain)</td>
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<tr>
<td>11:20~12:20</td>
<td>Constant mean curvature spacelike hypersurfaces in spacetimes with symmetries</td>
<td>Alfonso Romero (University of Granada, Spain)</td>
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12:20~14:00 Lunch Time

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<tr>
<td>14:00~14:45</td>
<td>Improved Chen-Ricci inequality for algebraic curvature tensors and its applications</td>
<td>Mukut Mani Tripathi (Banaras Hindu University, India)</td>
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14:45~15:00 Break Time

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<tr>
<td>15:00~15:30</td>
<td>On totally geodesic surfaces in $G_2/ SO(4)$</td>
<td>Misa Ohashi (Nagoya Institute of Technology, Japan)</td>
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<tr>
<td>15:30~16:00</td>
<td>Some topics related to Clifford algebras and the octonions</td>
<td>Hideya Hashimoto (Meijo University, Japan)</td>
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15:30~17:00 Discussion Time

17:00~17:10 Closing Remarks
Abstracts

July 26, 2016
(Tuesday)
In relation to Jacobi operator, we consider a new notion for real hypersurfaces in complex two-plane Grassmannians $G_2(\mathbb{C}^{m+2})$ and show results about real hypersurfaces in $G_2(\mathbb{C}^{m+2})$ with structure Jacobi operator and normal Jacobi operator under some condition.

**References**


The 20th International Workshop  
on Hermitian Symmetric Spaces and Submanifolds 
& The 12th RIRCM-OCAMI Joint Differential Geometry Workshop

Kyungpook National University, Daegu, Korea  
July 26(Tue) - 30(Sat), 2016

July 26(Tue), 2016

Hyunjin Lee*  
E-mail: lhjibis@hanmail.net  
(Research Institute of Real and Complex Manifolds, Kyungpook National University, Daegu 41566, India)

Young Jin Suh  
E-mail: yjsuh@knu.ac.kr  
(Department of Mathematics and RIRCM, Kyungpook National University, Daegu 41566, Korea)

1st 2

The parallelism for real hypersurfaces in complex Grassmannians of rank two

In this talk we study the cyclic parallel hypersurfaces in complex hyperbolic two-plane Grassmannians \( SU_{2,m}/S(U_2U_m) = G_2^∗(\mathbb{C}^{m+2}) \) which have a remarkable geometric structure as Hermitian symmetric spaces of rank 2. At first we prove that if the Reeb vector field belongs to the distribution \( Q^\perp = \text{span} \{ \xi_\nu | J_\nu N = -\xi_\nu, \ \nu = 1, 2, 3 \} \), then a cyclic parallel hypersurface \( M \) in \( G_2^∗(\mathbb{C}^{m+2}) \) must be Reeb parallel by virtue of the equation of Codazzi. Using this relation, we classify all cyclic (or Reeb, resp.) parallel hypersurfaces in \( G_2^∗(\mathbb{C}^{m+2}) \) with geodesic Reeb flow.

References

1st-3 Gyu Jong Kim*  E-mail: hb2107@naver.com  (Department of Mathematics, Kyungpook National University, Daegu 702-701, Korea)
Young Jin Suh  E-mail: yjsuh@knu.ac.kr  (Department of Mathematics, Kyungpook National University, Daegu 702-701, Korea)

Hypersurfaces in complex hyperbolic two-plane Grassmannians with GTW Reeb Lie derivative structure Jacobi operator

In this talk, we consider the complex hyperbolic two-plane Grassmannians $G_2^{n+2}(C)$ as the ambient space. Using previous results of Berndt and Suh, we give a existence theorem of a real hypersurface in complex hyperbolic two-plane Grassmannians related to a certain condition of the structure Jacobi operator.
Comparison on classification results of real hypersurfaces in compact and noncompact complex two-plane Grassmannians

In this talk, we introduce a new commuting condition between the structure (resp., normal) Jacobi operator and symmetric \((1,1)\)-type tensor field \(T\), that is, \(R_T\phi T = TR\phi\) (resp., \(R_N\phi T = TR_N\phi\)), where \(T = A\) or \(T = S\) for Hopf hypersurfaces in complex (hyperbolic) two-plane Grassmannians. By using simultaneous diagonalization for commuting symmetric operators, we give a complete classification of real hypersurfaces in complex (hyperbolic) two-plane Grassmannians with commuting condition respectively.

REFERENCES

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July 26(Tue) - 30(Sat), 2016

July 26(Tue), 2016

1st-5 Keomkyo Seo  E-mail: kseo@sookmyung.ac.kr
(Department of Mathematics, Sookmyung Women’s University, Seoul 04310, Korea)

Characterizations of a Clifford hypersurface in a unit sphere

The Clifford hypersurface is one of the simplest compact hypersurfaces in a unit sphere. In this talk, we give two different types of characterizations of Clifford hypersurfaces among constant m-th order mean curvature hypersurfaces with two distinct principal curvatures. One is obtained by assuming embeddedness and by comparing two distinct principal curvatures. The proof uses the maximum principle to the two-point function, which was used in the proof of Lawson conjecture by Brendle [1]. The other is given by obtaining a sharp curvature integral inequality for hypersurfaces in a unit sphere with constant m-th order mean curvature and with two distinct principal curvatures, which generalizes Simons integral inequality [4]. This talk is based on joint works [2, 3] with Sung-Hong Min.

References

On the pointwise slant submanifolds

In this talk, we consider several kinds of submanifolds in Riemannian manifolds, which are obtained by many authors. (i.e., slant submanifolds, pointwise slant submanifolds, semi-slant submanifolds, pointwise semi-slant submanifolds, pointwise almost h-slant submanifolds, pointwise almost h-semi-slant submanifolds, etc.) We investigate the properties of slant functions and semi-slant functions and also obtain the topological properties of proper pointwise slant submanifolds. We give an inequality for the squared norm of the second fundamental form in terms of a warping function and a slant function for a warped product submanifold of a Riemannian manifold. Finally, we give some examples of such submanifolds.
CR geometry on contact manifolds

For a contact manifold, we have two fundamental structures associated with the given contact form. One is a Riemannian structure and the other is a transversal almost complex structure. In this talk, we study pseudo-Hermitian geometry, which preserves a transversal almost complex structure.
Quaternionic projective spaces and 7-sphere

In this talk, we consider $U(n + 1)$ invariant polynomials on quaternionic projective spaces $\mathbb{HP}^n$ induced from an isoparametric function on sphere $S^{4n+3}$ via Hopf fibration, and discuss hypersurface geometry on the projective spaces $\mathbb{HP}^n$ and related spheres along the polynomials. In particular, we study the hypersurface geometry of 7-sphere $S^7$. 

Yong Seung Cho  
E-mail: yescho@ewha.ac.kr  
(Department of Mathematics, Ewha Womans University, Seoul 120-750, Korea)

Gromov-Witten invariants on the products of almost contact metric manifolds

We investigate Gromov-Witten invariants and quantum cohomologies on the products of almost contact metric manifolds. In case the manifolds are cosymplectic, then the product has a Kähler structure. On the products we get some geometric properties, Gromov-Witten invariants, and quantum cohomologies. We have some relations between Gromov-Witten invariants, quantum cohomologies of the products and the ones of two cosymplectic manifolds, respectively.

References

2nd-1 Eric Grinberg  E-mail: eric.grinberg@umb.edu
(Department of Mathematics, University of Massachusetts Boston, 100 Morrissey Blvd.
Boston, MA 02125-3393, USA)

Integral geometry along flat subvarieties in symmetric spaces of compact type

We consider the Radon transform that integrates over flat totally geodesic sub-
manifolds in a symmetric space of compact type. The salient theme is that either
the transform has a canonical kernel or else it is injective. We also explore variants
of this problem where ‘flat’ is replaced by ‘maximally curved’.
I will give talks on
(I) Introduction to the theory of isoparametric hypersurfaces
(II) Introduction to symplectic geometry and Lagrangian submanifolds
(III) Floer homology on intersections of Lagrangian submanifolds

Our results will be given by Ohnita’s talk.

REFERENCES


There is a well-known class of compact, complex, non-Kählerian manifolds constructed by Bosio, called the LVMB manifolds, which properly includes the Hopf manifold, the Calabi-Eckmann manifold, and the LVM manifolds. As in the case of LVM manifolds, these LVMB manifolds can admit a regular holomorphic foliation $\mathcal{F}$. Moreover, Meersseman shows that if an LVMB manifold is actually an LVM manifold, then the regular holomorphic foliation $\mathcal{F}$ is actually transverse Kähler.

The aim of this talk is to address the converse question. Namely, we want to discuss whether or not, when the holomorphic foliation $\mathcal{F}$ on an LVMB manifold $N$ is transverse Kähler, $N$ is actually an LVM manifold.
On Floer homology of the Gauss images of isoparametric hypersurfaces

The Gauss images of isoparametric hypersurfaces in the unit standard sphere provide compact minimal (thus monotone) Lagrangian submanifolds embedded in complex hyperquadrics $Q_n(\mathbb{C})$. Recently we used the Floer homology and the lifted Floer homology for monotone Lagrangian submanifolds in order to study their Hamiltonian non-displaceability in the joint paper [1]. In this talk, I would like to explain the spectral sequences for the Floer homology and the lifted Floer homology of monotone Lagrangian submanifolds and their applications to the Gauss images of isoparametric hypersurfaces, which are the main technical part in our joint work. Moreover I will suggest some related open problems.

REFERENCES

On Bonnesen-style inequalities for mixed volumes

The classical Bonnesen-style inequality measures the deficit of a convex body and a standard ball in the Euclidean space $\mathbb{R}^n$. The Alexandrov-Fenchel inequality for mixed volumes is the natural extension of the classical isoperimetric inequality. We may interested in the homotheties of convex bodies.
Quantitative rigidity on manifolds of Ricci curvature bound below I

This is the first of two talks under the same title. In Riemannian geometry, space forms (i.e., complete simply connected manifolds of constant sectional curvature) has been played an important role, serving as a model space and a source of ideas. In this talk, starting with two volume space form rigidity with lower Ricci curvature bound, we will discuss recent work on quantitative versions of the two volume space form rigidity. This work is joint with Lina Chen and Shicheng Xu of Capital Normal University.
The 20th International Workshop on
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Abstracts

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July 28(Thu), 2016

3rd-1 Eric Grinberg  E-mail: eric.grinberg@umb.edu
(Department of Mathematics, University of Massachusetts Boston, 100 Morrissey Blvd. Boston, MA 02125-3393, USA)

Integral geometry along flat subvarieties in symmetric spaces of compact type

We consider the Radon transform that integrates over flat totally geodesic submanifolds in a symmetric space of compact type. The salient theme is that either the transform has a canonical kernel or else it is injective. We also explore variants of this problem where ‘flat’ is replaced by ‘maximally curved’.
Reiko Miyaoka  
E-mail: r-miyaok@m.tohoku.ac.jp  
(Institute of Liberal Arts and Sciences, Tohoku University, Sendai 980-8576, Japan)

Hamiltonian non-displaceability of the Gauss images of isoparametric hypersurfaces (I)-(III)

I will give talks on
(I) Introduction to the theory of isoparametric hypersurfaces
(II) Introduction to symplectic geometry and Lagrangian submanifolds
(III) Floer homology on intersections of Lagrangian submanifolds

Our results will be given by Ohnita’s talk.

References

Alfonso Romero  
E-mail: aromero@ugr.es  
(Department of Geometry and Topology, University of Granada, 18071 Granada, Spain)

The classical Calabi-Bernstein theorem: history and new approaches

The classical Calabi-Bernstein theorem states that the only entire solutions of the maximal surface equation in 3-dimensional Lorentz-Minkowski spacetime are the affine functions defining spacelike planes. This relevant result was firstly proved by Calabi in 1970 and later extended to any dimension by Cheng and Yau in 1976. With this result, a new class of elliptic problems of geometric and physical relevance started. In this talk, several proofs of historical interest are reviewed and other new ones are explained, paying special attention to the different techniques used.
Chen-Nagano [1] introduced the notion of antipodal sets of compact Riemannian symmetric spaces. I showed a correspondence between maximal antipodal sets of oriented real Grassmann manifolds and certain families of subsets of finite sets and reduced the problem of the classifications of maximal antipodal sets of oriented real Grassmann manifolds to a combinatorial problem in [2]. Using this correspondence I showed some sequences of maximal antipodal sets in [3]. In this talk I construct new sequences of maximal antipodal sets from those obtained in [3].

References

Volume-preserving mean curvature flow for tubes in rank one symmetric spaces of non-compact type

First we state the evolutions of the radius function and its gradient along the volume-preserving mean curvature flow starting from a tube (of nonconstant radius) over a compact closed domain of a reflective submanifold in a symmetric space under certain condition for the radius function. Next, we prove that the tubeness is preserved along the flow in the case where the ambient space is a rank one symmetric space of non-compact type (other than a (real) hyperbolic space) and the reflective submanifold is an invariant submanifold and the radius function of the initial tube is radial. Furthermore, in this case, we prove that the flow reaches to the invariant submanifold or it exists in infinite time and converges to another tube of constant mean curvature over the invariant submanifold in the $C^1$-topology in infinite time.

References

Maximal antipodal subgroups of the automorphism groups of compact Lie algebras

The automorphism group $\text{Aut}(\mathfrak{g})$ of a comapct Lie algebra $\mathfrak{g}$ is a compact Lie group. A finite subgroup $A$ of a compact Lie group is an antipodal subgroup if $A$ is commutative and every element of $A$ is involutive. We classify maximal antipodal subgroups of $\text{Aut}(\mathfrak{g})$ when $\mathfrak{g}=\mathfrak{su}(n)$, $\mathfrak{o}(n)$, $\mathfrak{sp}(n)$, $G_2$. A maximal antipodal subgroup of $\text{Aut}(\mathfrak{g})$ gives us as many mutually commutative involutions of $\mathfrak{g}$ as possible. For the classification we use our former results of the classification of maximal antipodal subgroups of Lie groups $G = U(n)/\mathbb{Z}_2$, $SU(n)/\mathbb{Z}_2$, $O(n)/\{\pm 1\}$, $Sp(n)/\{\pm 1\}$, $G_2$ in [1, 2]. They contain the classification of maximal antipodal subgroups of the adjoint group $\text{Ad}(G)$ of $G$ which is isomorphic to the identity component of $\text{Aut}(\mathfrak{g})$ if the Lie algebra of $G$ is $\mathfrak{g}$. We also use canonical forms of elements in a compact Lie group which is not connected.

REFERENCES

Kazuyuki Hasegawa*  *E-mail: kazuhase@staff.kanazawa-u.ac.jp
(Faculty of teacher education, Institute of human and social sciences, Kanazawa university, Kakuma-machi, Kanazawa, Ishikawa, Japan)
Katsuhiro Moriya  *E-mail: moriya@math.tsukuba.ac.jp
(Division of Mathematics, Faculty of Pure and Applied Sciences, University of Tsukuba, 1-1-1 Tennodai, Tsukuba, Ibaraki, Japan)

Twistor lifts and factorization for conformal maps of a surface

The twistor theory serves an important role in the study of surfaces in four-dimensional Riemannian manifolds, in particular, minimal surfaces (see [1] and [3], for example). Quaternionic holomorphic geometry [2] is another useful theory for studying the surfaces in the special case. In this talk, we show explicitly a relation between the theory of twistor lifts and quaternionic holomorphic geometry. The use of twistor lifts has the advantage that it induces a factorization of the differential of a conformal map into a factor which describes intrinsic geometry of a surface and other factors which describe extrinsic geometry of a surface. More precisely, we show that the differential of a conformal map is factored by two maps into Sp(1) and a (1, 0)-form locally. We note that the (1, 0)-form gives the intrinsic invariant of a conformal map. The maps into Sp(1) give the generalized Gauss map of a surface. Using this factorization, we give a characterization for constrained Willmore surfaces etc.

REFERENCES

In 1983, J. Eells and L. Lemaire extended the notion of harmonic map between Riemannian manifolds to that of biharmonic map, which is defined as a critical point of the bienergy functional. G.Y. Jiang studied the first and second variation formulas of the bienergy functional and obtained the Euler-Lagrange equation, which is a fourth order PDE. In this talk, first we give a necessary and sufficient condition for a submanifold whose tension field is parallel to be biharmonic. For orbits of commutative Hermann actions in compact symmetric spaces, this condition can be described in terms of symmetric triads. By using this criterion, we determine all biharmonic hypersurfaces in irreducible symmetric spaces of compact type which are regular orbits of commutative Hermann actions of cohomogeneity one. Moreover, we construct higher codimensional proper biharmonic submanifolds in compact symmetric spaces and give some classification results. Here, we call a biharmonic map is proper if it is not harmonic.

REFERENCES

A duality between compact symmetric triads and semisimple pseudo-Riemannian symmetric pairs with applications to geometry of Hermann type actions

In this talk, we introduce the notion of a duality between commutative compact symmetric triads and semisimple pseudo-Riemannian symmetric pairs, which is a generalization of the duality between compact/noncompact Riemannian symmetric pairs. As its application, we study the action of a symmetric subgroup of $G$ on a pseudo-Riemannian symmetric space $G/H$, which is called a Hermann type action. We also give an alternative proof for Berger’s classification of semisimple pseudo-Riemannian symmetric pairs from a viewpoint of compact symmetric triads. This talk is based on a joint work with Osamu Ikawa (Kyoto Institute of Technology) and Atsumu Sasaki (Tokai University).
Quantitative space from rigidity under lower Ricci curvature bound II

This is the second of two talks under same title. In this talk, we will prove the quantitative volume space form rigidity conjecture under the additional condition: Ricci curvature is also bounded above. Using the upper bound on Ricci curvature enable, we will employ techniques of Ricci flow to show the existence of nearby metric of almost constant sectional curvature. This work is joint with Lina nChen and Shicheng Xu of Capital Normal University.
The 20th International Workshop on Hermitian Symmetric Spaces and Submanifolds & The 12th RIRCM-OCAMI Joint Differential Geometry Workshop

Kyungpook National University, Daegu, Korea
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July 29(Fri), 2016

4th-1 Eric Grinberg  
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Integral geometry along flat subvarieties in symmetric spaces of compact type

We consider the Radon transform that integrates over flat totally geodesic submanifolds in a symmetric space of compact type. The salient theme is that either the transform has a canonical kernel or else it is injective. We also explore variants of this problem where ‘flat’ is replaced by ‘maximally curved’.
4th-2  Reiko Miyaoka  E-mail: r-miyaok@m.tohoku.ac.jp  
(I Institute of Liberal Arts and Sciences, Tohoku University, Sendai 980-8576, Japan)

Hamiltonian non-displaceability of the Gauss images of isoparametric hypersurfaces (I)-(III)

I will give talks on
(I) Introduction to the theory of isoparametric hypersurfaces
(II) Introduction to symplectic geometry and Lagrangian submanifolds
(III) Floer homology on intersections of Lagrangian submanifolds

Our results will be given by Ohnita’s talk.

REFERENCES


Spacelike surfaces in GRW spacetimes whose mean curvature function satisfies a nonlinear inequality

A nonlinear inequality involving the mean curvature function of a spacelike surface and the restriction of the Hubble function in a 3-dimensional Generalized Robertson-Walker spacetime is considered and analyzed in detail. Under geometric assumptions, such complete spacelike surfaces are classified. In the parametric case, new Calabi-Bernstein type problems are then stated and solved as a consequence.
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4th-S1-4 Yasuo Matsushita  E-mail: yma24tak@gmail.com
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Almost complex structures on four-dimensionional neutral manifolds

The existence of a neutral metric ($+\ +\ -\ -$) on a 4-manifold is equivalent to the existence of a field of 2-planes, and also to the existence of two kinds of almost complex structures. Therefore, a neutral 4-manifold is necessarily an almost complex manifold, and simultaneously an opposite almost complex manifold. Concerning these almost complex structures, neutral 4-manifolds provides us various interesting problems, i.e., when they are integrable and when the corresponding Kähler forms are symplectic, and moreover when they are Kähler. Nontrivial examples of isotropic Kähler structures and a routine of producing non-flat neutral Kähler-Einstein 4-manifolds, and other interesting results are reported.
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4th-S2-4 Katsuhiro Moriya  E-mail: moriya@math.tsukuba.ac.jp
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The Schwarz lemma for super-conformal maps

A super-conformal map from a Riemann surface to the Euclidean four-space is a surface with circular ellipse of curvature with respect to the induced metric. This map has properties similar to the holomorphic function on a Riemann surface. In this talk, the Schwarz lemma for super-conformal maps is explained.
In this talk, we introduce the notion of Weyl semi-symmetric, projective semi-symmetric and conformal semi-symmetric curvature tensor defined on semi-Kaehler manifolds. Moreover, we give a complete classification of complex hypersurfaces $M$ in semi-Kaehler space forms $\mathbb{M}_{n+1}^+(c)$ with Weyl semi-symmetric, projective semi-symmetric or conformal semi-symmetric curvature tensor respectively and also investigate a relation of these conditions.
We study a kind of complex submanifolds in a quaternion projective space $\mathbb{H}P^n$, which we call transversally complex submanifolds, from the viewpoint of quaternionic differential geometry. There are several examples of transversally complex immersions of Hermitian symmetric spaces. For a transversally complex immersion $f : M \to \mathbb{H}P^n$, a key notion is a Gauss map associated with $f$, which is a map $S : M \to \text{End}(\mathbb{H}^{n+1})$ with $S^2 = -\text{id}$. Our theory is an attempt of a generalization of the theory "Conformal geometry of surfaces in $S^4$ and quaternions" by Burstall, Ferus, Leschke, Pedit, and Pinkall [1].

REFERENCES
On 3-dimensional real hypersurfaces in complex space forms

In this talk, we classify real hypersurfaces of a 2-dimensional complex space form whose Ricci operator $S$ and the structure vector field $\xi$ satisfy $S\xi = \beta\xi$ for some function $\beta$ and $g((\nabla_X S)Y, Z) = g((\nabla_Y S)X, Z)$, where $X, Y$ and $Z$ are perpendicular to $\xi$. And we consider some related conditions for the Ricci tensor $S$. 

A construction of weakly reflective submanifolds in compact symmetric spaces.

A submanifold of a Riemannian manifold is called a weakly reflective submanifold if for any point in the submanifold and each normal vector at the point, there exists a weakly reflection which is an isometry on the ambient Riemannian manifold. The notion of weakly reflective submanifold is a generalization of the notion of reflective submanifold. Reflective submanifolds in irreducible compact symmetric spaces are classified. In contrast, examples of weakly reflective submanifolds in compact symmetric spaces are not well known except for examples in hyperspheres. In this talk we will give examples of weakly reflective submanifolds in compact symmetric spaces as orbits of Hermann actions which are generalizations of isotropy actions of compact symmetric spaces. To construct weakly reflective submanifolds in compact symmetric spaces, we use symmetric triads which are generalizations of restricted root systems.
In 1889, Casorati ([1],[2]) defined a curvature (now well known as the Casorati curvature) which turns out to be the normalized sum of the squared principal curvatures. For a Riemannian submanifold the Casorati curvature is defined to be the normalized squared length of the second fundamental form [3].

The talk is organized as follows. An important Lemma for a convex optimization problem is presented. Given an $n$-dimensional Riemannian manifold $(M, g)$, a Riemannian vector bundle $(B, g_B)$ over $M$, a $B$-valued symmetric $(1, 2)$-tensor field $\zeta$ and a (curvature-like) tensor field $T$ satisfying the algebraic Gauss equation, we introduce the notion of different kind of algebraic Casorati curvatures $\hat{\delta}_{CT,B}(n-1)$, $\delta'_{CT,B}(n-1)$, $\delta_{CT,B}(r;n-1)$, $\hat{\delta}_{CT,B}(r;n-1)$, which in special cases of Riemannian submanifolds reduce to already known Casorati curvatures. We present results expressing Casorati inequalities for algebraic Casorati curvatures. Equality cases are also discussed. After this, application parts begin. We obtain basic inequalities for Casorati curvatures $\hat{\delta}(n-1)$, $\delta'(n-1)$, $\delta(n-1)$, $\delta(r;n-1)$, $\hat{\delta}(r;n-1)$ for Riemannian submanifolds. We further apply these results to obtain Casorati inequalities for Riemannian submanifolds of real space forms with a very short proof. We also present several results for Casorati ideal submanifolds/hypersurfaces of real space forms and Euclidean spaces. Finally, we present some problems for further studies.

References


Derivatives on real hypersurfaces of nonflat complex space forms

Let $M$ be a real hypersurface of a nonflat complex space form, that is, either a complex projective space or a complex hyperbolic space. On $M$ we have the Levi-Civita connection and for any nonnull real number $k$ the corresponding generalized Tanaka-Webster connection. Therefore on $M$ we consider their associated covariant derivatives, the Lie derivative and, for any nonnull $k$, the so called Lie derivative associated to the generalized Tanaka-Webster connection and introduce some classifications of real hypersurfaces in terms of the coincidence of some pairs of such derivatives when they are applied to the shape operator of the real hypersurface, the structure Jacobi operator, the Ricci operator or the Riemannian curvature tensor of the real hypersurface.
The role of some symmetries of spacetime which naturally arise in General Relativity is discussed. The importance of constant mean curvature (CMC) spacelike hypersurfaces in the study of the Einstein equation is recalled. In certain spacetimes with symmetry defined by a timelike conformal vector field or by a lightlike parallel vector field, uniqueness theorems of complete CMC spacelike hypersurfaces are given. In some cases, uniqueness results of Calabi-Bernstein type are obtained as applications.
Improved Chen-Ricci inequality for algebraic curvature tensors and its applications

In 1999, B.-Y. Chen [1, Theorem 4] proved a basic inequality involving the Ricci curvature and the squared mean curvature of submanifolds in a real space form. In [2], the author presented Chen-Ricci inequality and improved Chen-Ricci inequality for curvature like tensors and applied the results to study Kaehlerian slant submanifolds of complex space forms and Legendrian submanifolds of Sasakian space forms. In this talk, the author will give a short survey of results and some problems for further studies.

REFERENCES

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5th-4 Misa Ohashi E-mail: ohashi.misa@nitech.ac.jp
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On totally geodesic surfaces in $G_2/SO(4)$

This is a joint work with H. Hashimoto, K. Mashimo and F. Nakata. We explain
the Quaternionic Kähler structure on $G_2/SO(4)$. Recently, Professor Mashimo
obtained classification of totally geodesic surfaces in $G_2/SO(4)$. We give the ge-
ometrical properties (for example totally complex or totally real) of these totally
geodesic surfaces. We give the relationship between the totally geodesic surfaces in
$G_2/SO(4)$ and the 3-dimensional totally real submanifolds in 6-dimensional sphere
$S^6 = G_2/SU(3)$. 
Some topics related to Clifford algebras and the octonions

This is a joint work with K. Mashimo, F. Nakata and M. Ohashi. Let $\mathfrak{C}$ be the octonions or Cayley algebra and let $G_2$ be the group of automorphisms of $\mathfrak{C}$ with respect to the multiplication. There exist the set of homogeneous spaces related to $G_2$. For example, $G_2/SU(3)$ is isomorphic to the unit sphere $S^6$ in the purely imaginary octonions $\text{Im} \mathfrak{C}$ and $G_2/SO(4)$ can be considered as the Grassmann manifold of all associative 3-planes in $\text{Im} \mathfrak{C}$. Then there exists a twistor correspondence between the Nearly Kähler $S^6$ and Quaternionic Kähler $G_2/SO(4)$. The purpose of this talk is to give the fibre bundle structures related to the homogeneous spaces of $G_2$ and their geometrical structures. In particular, we want to describe the geometrical structures on the homogeneous spaces by using the invariant differential forms and moving frames of $G_2$. 